$$u_1=\left(\frac{6}{1}+\frac{6}{2}\right)-\left(\frac{6}{2}+\frac{6}{3}\right)-12.\frac{1}{2^2},$$

$$u_2=\left(\frac{6}{2}+\frac{6}{3}\right)-\left(\frac{6}{3}+\frac{6}{4}\right)-12.\frac{1}{3^2},$$

$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

$$u_{n-1}=\left(\frac{6}{n-1}+\frac{6}{n}\right)-\left(\frac{6}{n}+\frac{6}{n+1}\right)-12.\frac{1}{n^2}.$$

By addition, we have $\sum\limits_{r=1}^{r=n-1}u_r=9-\frac{6}{n}-\frac{6}{n+1}+12-12\sum\frac{1}{n^2}$. For the sum to infinity we have $\sum\limits_{r=1}^{r=\infty}u_r=21-12.\frac{\pi^2}{6}=21-2\pi^2$, on remembering that $\sum\limits_1^\infty\frac{1}{n^2}=\frac{\pi^2}{6}$.

Also solved by G. B. M. Zerr, J. W. Clawson, and H. V. Spunar.

**297. Proposed by W. J. GREENSTREET, Marling School, Stroud, England.**

If $a$, $b$, $c$, $d$, $f$, $g$, $h$ are all real, and $a$, $ab-h^2$, $abc+2fgh-af^2-bg^2-ch^2$ are all positive, show that $b$, $c$, $bc-f^2$, and $ca-g^2$ are also positive.

**I. Solution by C. R. MacINNIS, Princeton, N. J.**

Since both $a$ and $ab-h^2$ are positive, $b$ must be positive.

$$abc+2fgh-af^2-bg^2-ch^2\equiv\frac{(ab-h^2)(bc-f^2)-(hf-bg)^2}{b}.$$

Since the whole expression is positive and both $b$ and $ab-h^2$ are also positive, $bc-f^2>0$. Hence $c>0$. Similarly,

$$abc+2fgh-af^2-bg^2-ch^2\equiv\frac{(ab-h^2)(ca-g^2)-(hg-af)^2}{a},\text{ and }ca-g^2>0.$$

**II. Solution by A. F. CARPENTER, Hastings, Nebr.**

Since $ab-h^2$ is positive $ab>h^2$, and since $h$ is real, $h^2$ is positive. Then $ab$, which is greater than $h^2$, is positive. But $a$ is positive; hence $b$ is positive.

Now $b(abc+2fgh-af^2-bg^2-ch^2)=(ab-h^2)(bc-f^2)-(bg-fh)^2$; that is, $(bc-f^2)(ab-h^2)=b$ (a positive quantity)$+(bg-fh)^2=a$ positive quantity, and since $ab-h^2$ is positive, $(bc-f^2)$ is positive.

Again, $a(abc+2fgh-af^2-bg^2-ch^2)=(ab-h^2)(ca-g^2)-(af-hg)^2$, and it follows as before that $(ca-g^2)$ is positive.

From $bc-f^2$=a positive quantity, or $ca-g^2$=a positive quantity, it is readily seen that $c$ is positive.

Also solved by H. V. Spunar and G. B. M. Zerr.

## GEOMETRY.

330. Proposed by J. J. QUINN, Ph. D., New Castle, Pa.

A line pivoted at the origin revolving with a constant angular velocity, intersects another moving parallel to the $Y$-axis with a constant linear velocity. (1) Find the locus of their intersection when the ratio of their velocities is as $m:n$ referred to a quadrant and a radius, respectively. (2) Assume $m=3$ and $n=2$, and apply to the trisection of an angle. (3) Under what conditions will this curve become a quadratrix? (4) Name the curve.

Solution by H. V. SPUNAR, M. and E. E., East Pittsburg, Pa.

The angular velocity of the intersection point $P$, due to the rotation of the radius vector $\rho$, is constant, say $v_1=d\theta/dt$, and that linear due to the constant velocity of the moving line in the direction of $X$-axis is $v_2=dx/dt$.

(1) Assuming the ratio of the velocities $v_1/v_2=m/n$, the locus of the point $P$ is

$$\frac{m}{n}=\frac{dv}{dx}, \text{ or } X=\frac{n}{m}\theta+\theta_1.$$

Letting the starting point be the origin, we have in polar coordinates, $\rho\cos\theta=\frac{n}{m}\theta$.

The curve may be applied without any difficulty to the multisection as well as to the trisection of an angle.

(2) To apply the curve "$\rho\cos\theta=2(\theta/3)$" to the trisection of the given angle $\theta$. Draw $OP$ at an angle $\theta$ (*i. e.*, the angle to be trisected) and $PP_1$ perpendicular to $X$-axis; then $OP_1=\rho\cos\theta=2(\theta/3)=x_1$; since putting $\theta=3\phi$, $\rho\cos 3\phi=3[2(\theta/3)]=3x_1$. Trisecting the linear abscissae $OP_1=x_1$ and erecting at $Q_1(x_1/3)$, the perpendicular to the $X$-axis, cutting the curve at $Q$, then drawing $QO$, we obtain $\measuredangle QOX=\phi=(\theta/3)$.

(3) Let in $r\sin\phi=n\phi$ (Dinostrates' Quadratrix), $\phi=(\frac{1}{2}\pi-\theta)$=the complement of the angle $\theta$. Then $\rho\cos\theta=\frac{n}{m}\left(\frac{\pi}{2}-\theta\right)$.

(4) Hence the name of the curve may be Complementary Quadratrix.

Also solved very neatly by G. B. M. Zerr.

331. Proposed by C. N. SCHMALL, New York City.

The center of two spheres radii $r_1$, $r_2$, are at the extremities of a straight line $2a$ on which as a diameter a circle is described. Find a point on the circumference from which the greatest portion of spherical surface is visible.